Student Number

Hunters Hill High School

2022 **Trial Examination**

Mathematics Extension 1

General Instructions:

Reading time – 10 minutes
Working time – 2 hours
Write using black pen
NESA approved calculators may be used
A reference sheet is provided
For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 70

Section I – 10 marks (pages 3 – 7)
Attempt all Questions 1 – 10
Allow about 15 minutes for this section

Section II – 60 marks (pages 8 – 12)
Attempt all Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Question 1 – 10.

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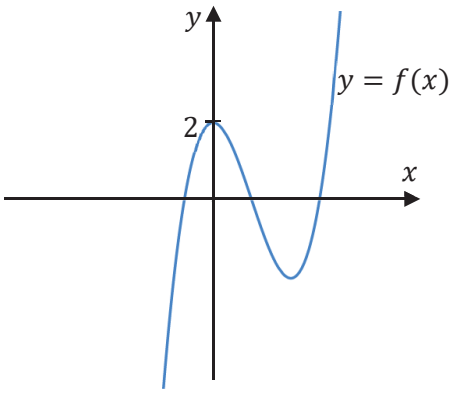
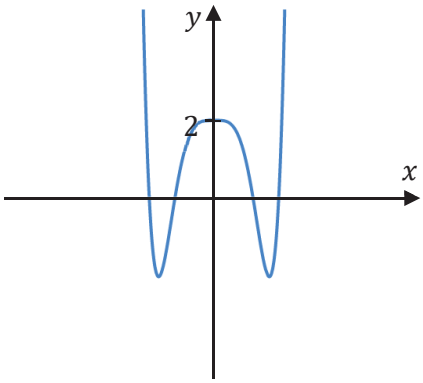
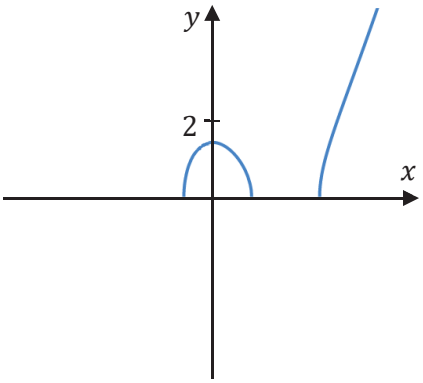
1. Given the sketch of $y = f(x)$ below, which of the following diagrams represents the sketch of $y^2 = f(x)$?



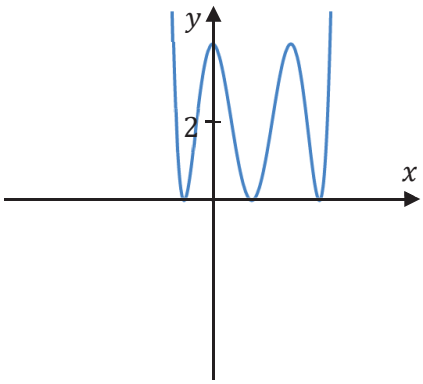
(A)



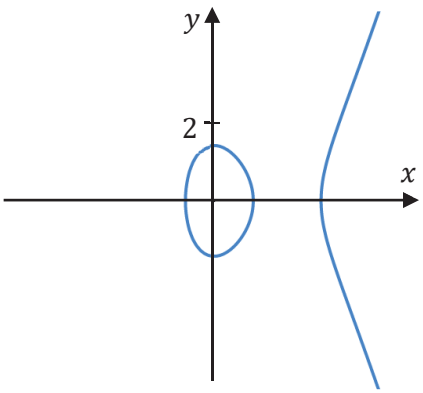
(B)



(C)



(D)



2. Given $R\sin(x+\alpha) = \sqrt{3}\sin x - \cos x$, where $R > 0$ and $-\pi \leq \alpha \leq \pi$, then:

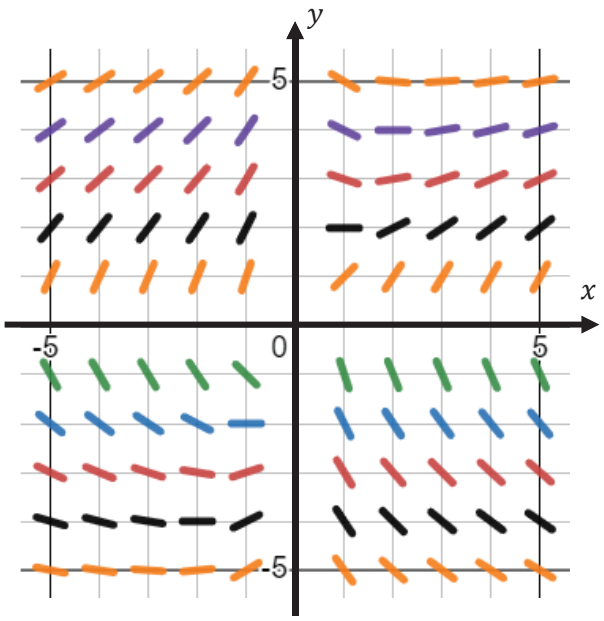
(A) $R = \sqrt{3}, \alpha = \dfrac{\pi}{6}$

(B) $R = \sqrt{3}, \alpha = -\dfrac{\pi}{6}$

(C) $R = 2, \alpha = \dfrac{\pi}{6}$

(D) $R = 2, \alpha = -\dfrac{\pi}{6}$

3.



The differential equation which best represents the above direction field is

(A) $\dfrac{dy}{dx} = \dfrac{2x+y}{xy}$

(B) $\dfrac{dy}{dx} = \dfrac{2x-y}{xy}$

(C) $\dfrac{dy}{dx} = \dfrac{2x-y}{x+y}$

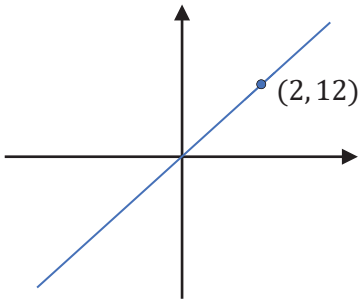
(D) $\dfrac{dy}{dx} = \dfrac{2x-y}{x-y}$

4. A function is defined by the parametric equations,

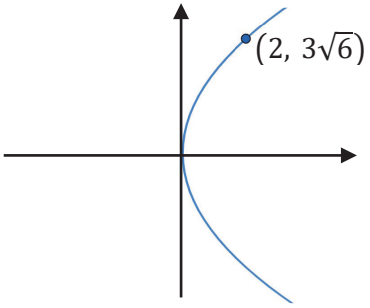
$$x = 3t^2$$
$$y = 9t, t > 0.$$

Which of the following sketches represents this function?

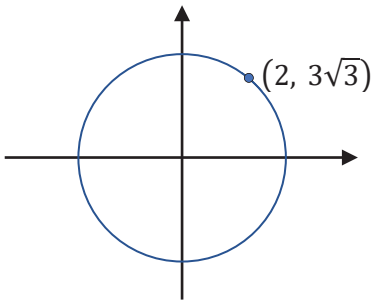
(A)



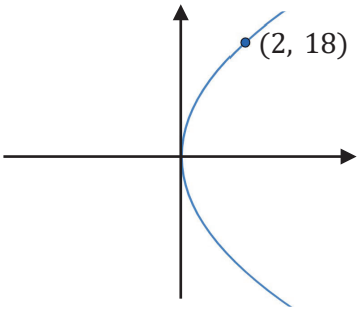
(B)



(C)



(D)



5. What is the domain of the function $f(x) = \cos^{-1}(3x)$?

(A) $-\pi \leq x \leq \pi$

(B) $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

(C) $-\frac{1}{3} \leq x \leq \frac{1}{3}$

(D) $-\frac{\pi}{9} \leq x \leq \frac{\pi}{9}$

6. The polynomial $P(x) = x^3 - 3x^2 - 7x + k$ has a remainder of 5 when divided by $x + 1$.
What is the value of k?

(A) -2

(B) 2

(C) -5

(D) 5

7. The angle between the vectors $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ is approximately

(A) $\cos^{-1}(0.99)$

(B) $\cos^{-1}(-0.99)$

(C) $\cos^{-1}(0.49)$

(D) $\cos^{-1}(-0.49)$

8. A spherical bubble is formed at the end of an air tube suspended within a gel. The bubble's radius increases at a rate of 0.1 mm/s.
At what rate is the surface area of the bubble increasing when it's radius reaches 7 mm, to the nearest mm/s?

(A) 18 mm/s

(B) 27 mm/s

(C) 56 mm/s

(D) 176 mm/s

9. In a lollypop factory the material for making each lollypop is sent through a machine.
The time, X seconds, taken to produce a lollypop by the machine is a binomial distribution where it can be shown that $P(X \leq 3) = \frac{5}{29}$.
A random sample of 12 lollypops is chosen. The probability that exactly 5 of these 12 lollypops took 3 or less seconds to produce, is:

(A) $\binom{12}{3}\left(\frac{5}{29}\right)^3\left(\frac{24}{29}\right)^{12}$

(B) $\binom{12}{3}\left(\frac{5}{29}\right)^3\left(\frac{24}{29}\right)^9$

(C) $\binom{12}{5}\left(\frac{5}{29}\right)^5\left(\frac{24}{29}\right)^{12}$

(D) $\binom{12}{5}\left(\frac{5}{29}\right)^5\left(\frac{24}{29}\right)^7$

10. The integral $\int \sin^2 4x\, dx$ is given by

(A) $\frac{1}{2}\left(x - \frac{1}{8}\sin 8x\right) + c$

(B) $\frac{1}{2}\left(x - \frac{1}{4}\sin 4x\right) + c$

(C) $\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + c$

(D) $\frac{1}{2}\left(\frac{x^2}{2} - \frac{1}{2}\sin 2x\right) + c$

End of Section I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Question 11 (15 marks)

a. Solve $\dfrac{4}{x+3} \geq x$. **3**

b. Prove that **2**

$$\frac{\cos 2\theta + \cos\theta + 1}{\sin 2\theta + \sin\theta} = \cot\theta$$

c. **i.** By expanding the left-hand side, show that **1**

$$\sin(5x + 4x) + \sin(5x - 4x) = 2\sin 5x \cos 4x.$$

ii. Hence find $\int \sin 5x \cos 4x \, dx$. **2**

d. A committee containing 3 men and 6 women is to be formed from a group of 12 men and 10 women.
In how many different ways can the committee be formed? **1**

e. A vat initially contains 400 grams of hydrogen chloride that is dissolved in 30 L of water, creating an acidic solution. A solution containing 25 grams of hydrogen chloride per litre of water is poured into the tank at a rate of 2 L per minute and the mixture in the tank is kept well stirred. At the same time, 3 L of the mixture flows out of the tank per minute.

Find the differential equation, $\dfrac{dm}{dt}$, where m represents the mass, in grams, of hydrogen chloride in the tank at time t minutes, for a non-zero volume of mixture. **2**

Question 11 continues on next page

f. The polynomial $P(x) = 2x^3 - x + 5$ has roots α, β and γ.

Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **2**

g. Two vectors are given by $\underset{\sim}{a} = 3\underset{\sim}{i} - m\underset{\sim}{j}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + n\underset{\sim}{j}$, where $m > 0$. **2**

If $\left|\underset{\sim}{a}\right| = 5$ and $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$, find the values of m and n.

End of Question 11

Question 12 – Begin a new writing booklet. (15 marks)

a. Find all values of θ, such that $-\pi \leq \theta \leq \pi$, that satisfy the equation **3**

$$3\sin\theta = \sqrt{3}\sin 2\theta.$$

b. Prove by mathematical induction that $2^{2n} + 15n - 1$ is divisible by 9 for $n \geq 1$. **3**

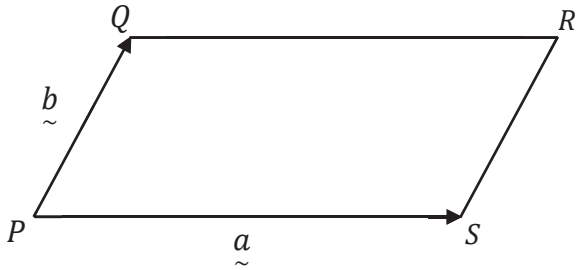
c. In an experiment, a pair of dice are rolled 80 times.
A success is recorded if the sum of the dice is 10 or more.

i. What is the mean of this binomial distribution? **3**
Give your answer to one decimal place.

ii. What is the standard deviation? **1**
Give your answer to one decimal place.

d. Let $P(x) = x^5 - 6x^3 - 8x^2 - 3x$. **2**
Show that $x = -1$ is a root of $P(x)$ of multiplicity three.

e. $PQRS$ is a parallelogram, where $\overrightarrow{PQ} = \underset{\sim}{a}$ and $\overrightarrow{PS} = \underset{\sim}{b}$. **3**



Prove, using vectors, that the sum of the squares of the lengths of the diagonals is equal to the sum of the squares of the lengths of the sides.

End of Question 12

Question 13 – Begin a new writing booklet. (15 marks)

a. The concentration of an active compound in a solution is $F(t)$, where t is the time in hours after the compound is added to the solution.
Initially, the concentration of the active compound is zero. The rate of change of concentration of the compound is given by

$$F'(t) = 40e^{-0.7t} - 0.6F(t).$$

i. By differentiating the product $F(t)e^{0.6t}$, show that **2**

$$\frac{d}{dt}(F(t)e^{0.6t}) = 40e^{-0.1t}.$$

ii. Hence, or otherwise, show that $F(t) = 400(e^{-0.6t} - e^{-0.7t})$. **2**

iii. The concentration of the active compound is increasing to a maximum.
For what value of t does this maximum occur? **2**

b. **i.** Find $\frac{d}{d\theta}(\sin^3\theta)$. **1**

ii. Use the substitution $x = \tan\theta$ to evaluate $\int_0^1 \frac{x^2}{(1+x^2)^{\frac{5}{2}}}dx$. **4**

c. **i.** Use the binomial theorem to find an expression for the constant term in the expansion of $\left(2x^3 - \frac{1}{x}\right)^{12}$. **3**

ii. For what values of n does $\left(2x^3 - \frac{1}{x}\right)^n$ have a non-zero constant term? **1**

End of Question 13

Question 14 – Begin a new writing booklet. (15 marks)

a. A phone manufacturer knows that 4% of the mobile phones it produces will be defective.
Phones are shipped in boxes of 20 phones. Boxes are shipped on pallets containing 200 boxes.

Calculate the probability that

i. A box of mobile phones contains exactly 3 defective phones. **1**

ii. A box of mobile phones contains at least 1 defective phone. **1**

iii. A pallet contains between 90 and 95 (inclusive) boxes with at least 1 defective phone. **3**
Use the probability table provided.

b. Find the general solution of $\frac{dy}{dx} = e^{y-x}$. **2**

c. At a carnival, an air cannon fires a t-shirt at time $t = 0$ seconds from an origin O at ground level across a level field.
The position vector $\underset{\sim}{r}(t)$, from O, of the t-shirt after t seconds is given by

$$\underset{\sim}{r}(t) = 15\sqrt{3}t\,\underset{\sim}{i} + (15t - 4.9t^2)\underset{\sim}{j},$$

where $\underset{\sim}{i}$ is a unit vector in the forward direction, $\underset{\sim}{j}$ is a unit vector vertically up and displacement components are measured in metres.

i. Find the initial velocity of the t-shirt and the initial angle, in degrees, of its trajectory to the horizontal. **2**

ii. Find the maximum height reached by the t-shirt, giving your answer in metres, correct to two decimal places. **2**

iii. Find the time of flight of the t-shirt. Give your answer in seconds, correct to three decimal places. **1**

iv. Find the range of the t-shirt in metres, correct to one decimal place. **1**

v. A person in the crowd, more than 40 m from O, catches the t-shirt at a height of 2 m above the ground.
How far horizontally from O is this person when the t-shirt is caught?
Give your answer in metres, correct to one decimal place. **2**

End of Examination

Probability Table for Question 14a

z	+ 0	+ 0.01	+ 0.02	+ 0.03	+ 0.04	+ 0.05	+ 0.06	+ 0.07	+ 0.08	+ 0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

MATHEMATICS EXTENSION 1
TRIAL Examination SOLUTIONS and MARKING GUIDELINES

Q1. If $y^2 = f(x)$
then $y = \pm\sqrt{f(x)}$

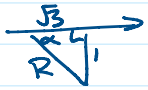
B represents the positive root.

so **D**

Q2 $R\sin(x+\alpha) = \sqrt{3}\sin x - \cos x$

$R\sin x\cos\alpha + R\cos x\sin\alpha = \sqrt{3}\sin x - \cos x$

$\therefore R\cos\alpha = \sqrt{3}$
$R\sin\alpha = -1$ $\Rightarrow$



$R = 2, \quad \alpha = -\frac{\pi}{6}$ **D**

Q3. test $(1,2)$ $\frac{dy}{dx} = 0 \Rightarrow$ ~~A~~

test $(1,1)$ $\frac{dy}{dx} \div 1 \Rightarrow$ ~~D~~ ~~C~~

B

Q4. for $x = 2$, $2 = 3t^2$

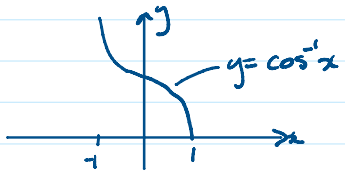
$t = \sqrt{\frac{2}{3}}$

$y = 9\sqrt{\frac{2}{3}}$

$= \frac{9\sqrt{6}}{3}$

$= 3\sqrt{2}$ **B**

Q5.



$x \Rightarrow 3x$ reduces domain by factor of 3.

C

Q6. $P(x)=x^3-3x^2-7x+k$

$P(-1)=5$

$(-1)^3-3(-1)^2-7(-1)+k=5$

$-1-3+7+k=5$

$k=2.$ B

Q7. $\cos\theta=\dfrac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|}$

$=\dfrac{2(-1)+3(2)}{\sqrt{2^2+3^2}\cdot\sqrt{(-1)^2+2^2}}$

$=\dfrac{4}{\sqrt{13}\cdot\sqrt{5}}$

$\theta\doteq\cos^{-1}(0496)$ C

Q8. $\dfrac{dr}{dt}=0.1$ $S=4\pi r^2$

$\dfrac{dS}{dr}=8\pi r.$

$\dfrac{dS}{dt}=\dfrac{dS}{dr}\cdot\dfrac{dr}{dt}$

$=\dfrac{8\pi r}{10}$

at $r=7$

$\dfrac{dS}{dt}=\dfrac{8\pi}{10}\cdot 7$

$=5.6\pi$ A

Q9. $P(X\leq 3)=\dfrac{5}{25}\Rightarrow p$

$q=1-p$

$=\dfrac{21}{25}.$ D

Q10. $\cos 2x=1-2\sin^2 x$

$8x$ $4x$

$\int\sin^2 4x\,dx=\frac{1}{2}\int(1-2\cos 8x)\,dx$ A.

Question 11

Friday, 19 August 2022 7:40 AM

Q11

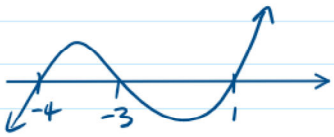
a. $\frac{4}{x+3} \geq x \qquad x \neq -3$

$4(x+3) \geq x(x+3)^2$

$(x+3)\left[x(x+3)-4\right] \leq 0$

$(x+3)(x^2+3x-4) \leq 0$

$(x+3)(x+4)(x-1) \leq 0$



$\therefore x \leq -4, -3 < x \leq 1$ is the solution

1– multiplies by $(x+3)^2$

2– correct factors

3– correct solution

b. Prove $\frac{\cos 2\theta + \cos\theta + 1}{\sin 2\theta + \sin\theta} = \cot\theta$

$LHS = \frac{2\cos^2\theta - 1 + \cos\theta + 1}{2\sin\theta\cos\theta + \sin\theta}$

$= \frac{\cos\theta(2\cos\theta + 1)}{\sin\theta(2\cos\theta + 1)}$

$= \cot\theta$

$= RHS$

1– uses one of the double angle identities correctly

2– correctly shown

c. i. $LHS = \sin(5x+4x) + \sin(5x-4x)$

$= \sin 5x\cos 4x + \cos 5x \sin 4x + \sin 5x \cos 4x - \cos 5x \sin 4x$

$= 2\sin 5x \cos 4x$

1– correctly shown

ii. $\int \sin 5x \cos 4x\,dx = \frac{1}{2}\int 2\sin 5x \cos 4x\,dx$

$= \frac{1}{2}\int (\sin 9x + \sin x)\,dx$

$= -\frac{1}{2}\left(\frac{\cos 9x}{9} + \cos x\right) + C$

1– correct integrand from i

2– correct answer (ignore +C)

d. choose 3 men from 12 $\Rightarrow {}^{12}C_3$

choose 6 women from 10 $\Rightarrow {}^{10}C_6$

number of ways $= {}^{12}C_3 \times {}^{10}C_6$

$= 46200$

1– correct answer

$= 46200$

1– correct answer

e. $V(t) = 400 + (2-3)t$
$= 400 - t$

input $25\text{g/L} \times 2\text{L/min} \Rightarrow \frac{dm}{dt}_{in} = 50\text{ g/min}$

output $3\text{L/min} \times n\text{ g/L}$ $\quad \frac{dm}{dt}_{out} = 3 \times \frac{n}{400-t}$
$= \frac{3n}{400-t}$

$\therefore \frac{dn}{dt} = 50 - \frac{3n}{400-t}$

1– finds expression for $V(t)$, input or output

2– correct expression

f. $P(x) = 2x^3 - x + 5$

$\sum \alpha\beta = -\frac{1}{2}$
$\sum \alpha\beta\gamma = -\frac{5}{2}$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$
$= -\frac{1}{2} \div \frac{5}{2}$
$= \frac{1}{5}$

1.– rewrites expression

2– correct ans.

g. $\underset{\sim}{a} = 3\underset{\sim}{i} - n\underset{\sim}{j}$ $\quad \underset{\sim}{b} = -2\underset{\sim}{i} + n\underset{\sim}{j}$

$|\underset{\sim}{a}| = 5$
$\therefore \sqrt{3^2 + n^2} = 5$
$n^2 = 16$
$n = 4$

$\underset{\sim}{a} \perp \underset{\sim}{b} \Rightarrow \underset{\sim}{a} \cdot \underset{\sim}{b} = 0$
$\therefore 3(-2) - 4(n) = 0$
$4n = -6$
$n = -\frac{3}{2}$

$\therefore m = 4,\ n = -\frac{3}{2}$

1– finds m

2– finds n.

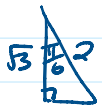
Q12

a. $3\sin\theta = \sqrt{3}\sin 2\theta$

$= 2\sqrt{3}\sin\theta\cos\theta$

$\sin\theta(3 - 2\sqrt{3}\cos\theta) = 0$

$\therefore \sin\theta = 0, \quad \cos\theta = \frac{\sqrt{3}}{2}$



$\theta = -\pi, 0, \pi, \frac{\pi}{6}, -\frac{\pi}{6}$

1 – use double angle formula
2 – factorises
3 – correct answers

b. Prove $2^{2n} + 15n - 1$ is divisible by 9.

Prove true for $n = 1$

$2^{2(1)} + 15(1) - 1 \rightarrow 4 + 15 - 1$

$= 18$ which is divisible by 9.

$\therefore$ true for $n = 1$

1 – proves base case

Assume true for $n = k$

$\therefore 2^{2k} + 15k - 1 = 9M$ (M is an integer)

– induction hypothesis (*)

Prove true for $n = k+1$

$2^{2(k+1)} + 15(k+1) - 1 = 2^2 \cdot 2^{2k} + 15k + 14$

$= 4 \cdot 4^k + 15k + 14$

$= 4(9M - 15k + 1) + 15k + 14$ by *

$= 9(4M) - 45k + 18$

$= 9(4M - 5k + 2)$

which is divisible by k

2 – uses induction hypothesis correctly

$\therefore$ by principle of induction $2^{2n} + 15n - 1$ is divisible by 9.

3 – correct proof

c. i. $\overline{|} \ldots 456$

c. i.

	... 4 5 6
⋮	
4	10
5	10 11
6	10 11 12

let P(10 or more) $= \frac{6}{36}$ $n = 80$

$= \frac{1}{6}$

1– determines P(10 or more)

Let X = number of rolls ≥ 10

$X \sim Bin\left(80, \frac{1}{6}\right)$

2– correct / not rounded

$E(X) = np$

$= 80 \times \frac{1}{6}$

$= 13.3$ (1dp)

3– correct to 1dp

ii. $Var(X) = np(1-p)$

$\sigma^2 = 80 \cdot \frac{1}{6} \cdot \frac{5}{6}$

$= 11.1$

$\therefore \sigma = 3.3$ (1dp)

1– correct ans.

d. $P(x) = x^5 - 6x^3 - 8x^2 - 3x$.

$P'(x) = 5x^4 - 18x^2 - 16x - 3$

$P''(x) = 20x^3 - 36x - 16$

$P(-1) = (-1)^5 - 6(-1)^3 - 8(-1)^2 - 3(-1)$

$= -1 + 6 - 8 + 3$

$= 0$

1– shows $P(-1) = 0$

$P'(-1) = 5(-1)^4 - 18(-1)^2 - 16(-1) - 3$

$= 5 - 18 + 16 - 3$

$= 0$

$P''(-1) = 20(-1)^3 - 36(-1) - 16$

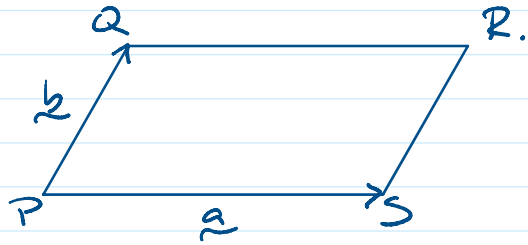
$= -20 + 36 - 16$

$= 0$

as $P(-1) = P'(-1) = P''(-1) = 0$, then $x = -1$ is a root of multiplicity three.

2– Shows $P(-1) = P'(-1) = P''(-1) = 0$

e.



$\overrightarrow{QR} = \overrightarrow{PS} = \underset{\sim}{a}$.

$\overrightarrow{SR} = \overrightarrow{PQ} = \underset{\sim}{b}$

diagonals: $\overrightarrow{PR} = \underset{\sim}{a} + \underset{\sim}{b}$

$\overrightarrow{QS} = \underset{\sim}{a} - \underset{\sim}{b}$..

$$|\underset{\sim}{a}+\underset{\sim}{b}|^2 + |\underset{\sim}{a}-\underset{\sim}{b}|^2 = (\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b}) + (\underset{\sim}{a}-\underset{\sim}{b})(\underset{\sim}{a}-\underset{\sim}{b})$$

$$= \underset{\sim}{a}\cdot\underset{\sim}{a} + 2\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{b} + \underset{\sim}{a}\cdot\underset{\sim}{a} - 2\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{b}$$

$$= |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2$$

which is the sum of the squares of the sides

1- $|\underset{\sim}{x}|^2 \Rightarrow \underset{\sim}{x}\cdot\underset{\sim}{x}$

2- expansions correct

3- correct proof.

Q13.

a.i $\frac{d}{dt}\left(F(t)e^{0.6t}\right) = F'(t)e^{0.6t} + 0.6F(t)e^{0.6t}$

$= \left(40e^{-0.1t} - 0.6F(t)\right)e^{0.6t} + 0.6F(t)e^{0.6t}$

$= 40e^{-0.1t} - 0.6F(t)e^{0.6t} + 0.6F(t)e^{0.6t}$

$= 40e^{-0.1t}$

1 – product rule to differentiate

2 – correctly shown

ii. separating variables

$\int d\left(F(t)e^{0.6t}\right) = \int 40e^{-0.1t}\,dt$

$F(t)e^{0.6t} = -400e^{-0.1t} + c$

as $F(0) = 0$, $c = 400$

$F(t) = \frac{400(1 - e^{-0.1t})}{e^{0.6t}}$

$= 400\left(e^{0.6t} - e^{-0.7t}\right)$

1 – separates variables

2 – answer shown

iii. Maximum when $F'(t) = 0$

$F'(t) = 400\left(-0.6e^{-0.6t} + 0.7e^{-0.7t}\right)$

$\therefore -0.6e^{-0.6t} + 0.7e^{-0.7t} = 0$

$-0.6 + 0.7e^{0.1t} = 0$

$e^{-0.1t} = \frac{6}{7}$

$-0.1t = \ln\frac{6}{7}$

$t = -10\ln\frac{6}{7}$

$= 1.5415$

1 – Solves for $F'(t) = 0$

2 – correct time

b. i. $\frac{d}{d\theta}(\sin^3\theta) = 3\cos\theta \cdot \sin^2\theta$

1 – correct derivative

!! let $x = \tan\theta$
$dx = \sec^2\theta\, d\theta$

at $x=0,\ \theta=0$
$x=1,\ \theta=\frac{\pi}{4}$

$$\therefore \int_0^1 \frac{x^2}{(1+x^2)^{\frac{5}{2}}}dx = \int_0^{\frac{\pi}{4}} \frac{\tan^2\theta}{(1+\tan^2\theta)^{\frac{5}{2}}}\cdot \sec^2\theta\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{\tan^2\theta}{\sec^5\theta}\sec^2\theta\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{\tan^2\theta}{\sec^3\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^3\theta\, d\theta$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{4}} 3\sin^2\theta\cos\theta\, d\theta$$

$$= \frac{1}{3}\left[\sin^3\theta\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{3}\left(\sin^3\frac{\pi}{4} - \sin^3 0\right)$$

$$= \frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)^3$$

$$= \frac{1}{6\sqrt{2}}$$

1-2 correct dx and bounds

3- correct substitution

4.- ans

C. i. $$\left(2x^3 - \frac{1}{x}\right)^{12} = \sum_{r=0}^{12}\binom{12}{r}(2x^3)^r(-x^{-1})^{12-r}$$

for constant $x^{3r}\cdot x^{-(12-r)} = x^0$
so $3r + (r-12) = 0$
$4r = 12$
$r = 3$

$\therefore$ constant term is $\binom{12}{3}2^3(-1)^9$
$= -1760$

1- expresses in sigma notation

2- equates powers

3- correct answer

ii. $x^{3r}\cdot(x^{-1})^{n-r} = x^0$

$3r - n + r = 0$
$r = \frac{n}{3}$

$\therefore \left(2x^3 - \frac{1}{x}\right)^n$ will have a non-zero constant when n is divisible by 3

1- correct answer

Question 14

Q14a. i Let p be the probability that a phone is defective $\Rightarrow p = 0.04$

Let X be the number of defective phones

$X \sim Bin(20, 0.04)$

$P(X=3) = {}^{20}C_3 (0.04)^3 (0.96)^{17}$

$= 0.0364499$

1 – correct ans

ii $P(X \geq 1) = 1 - P(X=0)$

$= 1 - {}^{20}C_0 (0.04)^0 (0.96)^{20}$

$= 0.557998$

1 – correct ans

iii $n = 209 \quad p = 0.558$

$\mu = np$

$= 111.6$

1 – correct σ

$\sigma = \sqrt{np(1-p)}$

$= 7.02$

$P(90 \leq X \leq 95) = P(-3.08 \leq Z \leq -2.36)$

2 – a correct zscore

$= P(Z \leq 3.08) - P(Z \leq 2.36)$

$= 0.9990 - 0.9909$

$= 0.0081$

3 – ans.

b. $\frac{dy}{dx} = e^{y-x}$

$= \frac{e^y}{e^x}$

$\int e^{-y} dy = \int e^{-x} dx$

1 – separates variables

$-e^{-y} = -e^{-x} + c$

$-e^{y} = -e^{-x} + c$

$e^{-y} = e^{-x} + k.$

$-y = \ln(e^{-x} + k)$

$y = \ln\left(\frac{e^{x}}{1 + ke^{x}}\right)$

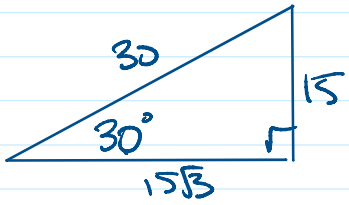
2 - makes y the subject.

c. i $\underset{\sim}{r}(t) = 15\sqrt{3}\,\underset{\sim}{i} + (15t - 4.9t^2)\,\underset{\sim}{j}$

$\underset{\sim}{\dot{r}}(t) = 15\sqrt{3}\,\underset{\sim}{i} + (15 - 9.8t)\,\underset{\sim}{j}.$

$\underset{\sim}{\dot{r}}(0) = 15\sqrt{3}\,\underset{\sim}{i} + 15\,\underset{\sim}{j}.$

1 - correct expr. for velocity or equiv.



initial velocity is $30\ ms^{-1}$, with an angle of $30°$.

2 - correct ans

ii. max height when $\dot{y} = 0$

vertical velocity $\dot{y} = 15 - 9.8t$

$\Rightarrow t = 1.5306$

1 - time of max height

$y = 15t - 4.9t^2$

$= 15(1.5306) - 4.9(1.5306)^2$

$= 11.47959$

$= 11.48\ m$

2 - correct ans

iii time of flight when $y = 0$

$15t - 4.9t^2 = 0$

$+(15 - 4.9t) = 0$

$\therefore t = 3.061\ sec$

1 - correct ans

iv. at $t=3.061$,

$x = 15\sqrt{3}(3.061)$

$= 79.52711$

$= 79.5\ m$

1 – correct ans

v. $y = 2$

$15t - 4.9t^2 = 2$

$4.9t^2 - 15t + 2 = 0$

$t = \dfrac{15 \pm \sqrt{125 - 4(49)2}}{2(49)}$

$= \dfrac{15 + \sqrt{85.8}}{9.8}$

$= 2.4758\ sec$

1 – finds correct quadratic

$x = 15\sqrt{3} \times 2.4758$

$= 64.323$

$= 64.3\ m.$

2 – correct ans